

Table of Contents

UNIT SYNOPSIS.....	2
CONTENT STANDARDS.....	3
LEARNING SUPPORTS BY LESSON.....	4
The EFFL Model.....	5
ROADMAP.....	6
UNPACKED STANDARDS.....	17
VERTICAL STANDARDS.....	21

How to Use This Addenda

Make sure you're ready to teach by noting the **Necessary Materials and Pre-Lesson Prep** you will need to gather or complete prior to the lesson

Find high-leverage instructional moves in the **Lesson Look Fors**. This is what leaders should see when observing your instruction

Note how your lesson objective ties to your state **Standards**

Plan purposeful questioning and responses using **Opportunities to CFU**

Plan to stress **Important Vocabulary** in the lesson. New vocab for the unit is indicated in bold

Note exemplar **pacing in the Lesson Agenda**

Use the **Mathematical Goal of the Lesson** to keep you focused on the appropriate student outcome

Plan instruction around what students need to Know & Do to be successful on the Exit Ticket using the identified **Student Know/Do Chart**

Find recommended lesson modifications, content knowledge boosters, and/or high-leverage instructional moves that may not be in your Teacher Edition located in **Other Notes to Inform Your Planning**

Date: _____

Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models

Standard(s): **3.4K** solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects; pictorial models, including arrays, area models; and equal groups; properties of operations; or recall of facts

Notes for Intellectual Preparation: Lesson Planning

Necessary Materials and Pre-Lesson Prep

- (S) Multiply by 2 (1–5) Pattern Sheet
- (S) Personal white board
- (S) Threes array no fill template
- (S) Blank paper

Lesson Agenda	Time
I. Do Now (source: fluency #1)	5 min
II. Fluency*	8 min
III. Concept Development	25 min
IV. Student Practice	15 min
V. Student Debrief	7 min
VI. Exit Ticket*	5 min

Mathematical Goal of this Lesson
Students learn they can use decomposition to break one larger number into two smaller numbers as a strategy for multiplication. The goal of this lesson is simply for student to understand how to interpret and create an array that demonstrates such decomposition. Students will build on this understanding in subsequent lessons. This lesson also supports the goal of student thinking in terms of counting units, an overarching goal for academy math.

Opportunities to CFU

- ✓ Concept Development, by way of eliciting student responses
- ✓ Problems Set problems: #2, #3

Important Vocabulary

- array
- **bracket**
- columns
- rows
- unit(s)

In this lesson, students are NOT responsible for the vocabulary distributive property. Please withhold as it will come up in later lessons.

Other Notes to Inform Your Planning

For Do Now: Use the Multiply by 2 (1–5) Pattern Sheet for your Do Now. 3 minutes for completion, 2 minutes whole group classwork check.

For Fluency: Complete the Group Counting activity (notice the inclusion of 4s in preparation for upcoming lessons) and Forms of Multiplication activity.

For Concept Development: Consider prepping personal whiteboard in advance. Spend no more than 12 minutes for CD Problem 1 and 13 minutes for CD Prob 2.

For Student Practice: consider creating an extra set of Qs like 1-3 in case students struggle with entry-level understanding. If they don't, move on to Qs 4 and above.

For Student Debrief: consider using the Eureka assigned Exit Ticket for whole group debrief exercise; Suggested strategy – guided discourse.

For Exit Ticket: Use Homework problems 2 & 3 for this lesson's Exit Ticket.

Other Notes: Though not formally discussed yet, this is a foundation to understanding of distributive property. Students visually see multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together.

$7 \text{ threes} = 5 \text{ threes} + 2 \text{ threes}$
 $7 \times 3 = 5 \times 3 + 2 \times 3$
 $21 = 15 + 6$

Lesson Look Fors

Look for teachers to...

- Have established a signaling routine for choral response or work show during the respective fluency activities
- Use a think aloud to describe why they shade what portions of the array, or use a different symbol in the array
- Make the focus of the lesson understanding the visual representations

Look for students to...

- Explain what they see in the array and how it relates to a given number sentence.

Student Criteria for Success

- Shading, brackets, and/or dotted lines on an array will have mathematical significance
- brackets can identify parts or wholes
- dotted lines and shading represent decompositions
- We count units; In an array, counting rows is the same as counting units.
- Addition/subtraction and multiplication math facts (up to 4)
- Interpret an array
- identify decompositions within an array
- Relate an annotated or labeled array to one or more number sentences
- Addition/subtraction (+/- up to 4)
- Multiplication (2, 3, and 4)

UNIT SYNOPSIS

Students graph the function $f(x) = \frac{1}{x}$, and analyze the key attributes such as domain/range, intercepts, symmetries, and asymptotic behavior, determining the asymptotic restrictions on the domain of a rational function and representing domain and range using interval notation, inequalities, and set notation. Students analyze the effect on the graph of $f(x) = \frac{1}{x}$, when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a , b , c , and d . Students investigate parameter changes and key attributes in terms of real-world problem situations. Students determine the sum, difference, product, and quotient of rational expressions with integral exponents of degree one and of degree two. Students solve rational equations that have real solutions and determine the reasonableness of the solutions. In real-world situations, students formulate rational equations, including inverse variation equations, solve equations, and justify solution(s) in terms of the problem situation.

Functions: A function is a relationship between variables in which each value of the input variable is associated with a unique value of the output variable. Functions can be represented in a variety of ways, such as graphs, tables, equations, or words. Each representation is particularly useful in certain situations. Some important families of functions are developed through transformations of the simplest form of the function.

- o Transformations of the parent reciprocal function include stretches, compressions (or shrinks), reflections, and horizontal and vertical translations.
- o A rational function is a ratio of polynomial functions. If a function has a polynomial in its denominator, its graph has a gap at each polynomial zero. The gap could be a one-point hole in the graph, or it could be the location of a vertical asymptote for the graph. You can get a reasonable graph for a rational function by finding all intercepts and asymptotes.

Equivalence: A single quantity may be represented by many different expressions.

The facts about a quantity may be expressed in many different equations.

- o You can use much of what you know about multiplying and dividing fractions to multiply and divide rational expressions.
- o To operate with rational expressions, you can use much of what you know about operating with fractions.
- o To solve an equation containing rational expressions, first multiply each side by the least common denominator of the rational expressions. This may introduce extraneous solutions.

Misconceptions:

- Some students may think an asymptote is a line that can never be crossed rather than a line that is approached. Although a vertical asymptote cannot be crossed, a horizontal asymptote can be crossed and approached in another section of the graph.
- Some students may think that when zeros of an expression occur in the denominator of the function, it always produces a vertical asymptote rather than understanding that if an x -value makes both the numerator and the denominator equal to zero, it indicates a removable discontinuity, not a vertical asymptote.
- Some students may have misconceptions about the nature of the graph of a function when using a graphing calculator, rather than understanding that sometimes the calculator obscures the details or hidden behavior of a function.

Underdeveloped Concepts:

- Some students struggle with operations of fractions and will need to be reminded of the traditional algorithm to translate to operations of rational expressions.

Key Questions:

- Rational functions can be written as ratios of two polynomial functions, have rates of change that are influenced by the polynomial functions within these ratios, and can be used to describe, model, and make predictions about situations.
- What graphs, key attributes, and characteristics are unique to rational functions?
- What are the real-world meanings of the key attributes of rational function models?
- How can the key attributes of rational functions be used to make predictions and critical judgments?
- What relationships exist between the algebraic forms of a rational function and the graph and key attributes of the function?

CONTENT STANDARDS

Below are the standards addressed in this unit.

Readiness Standards	Supporting Standards
<p>2A.2A Graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^2$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = x$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.</p>	<p>2A.6G Analyze the effect on the graph of $f(x) = 1/x$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d.</p>
<p>2A.6I Solve rational equations that have real solutions.</p>	<p>2A.6H Formulate rational equations that model real-world situations.</p>
<p>2A.6L Formulate and solve equations involving inverse variation (and combined variation).</p>	<p>2A.6J Determine the reasonableness of a solution to a rational equation.</p>
<p>2A.7F Determine the sum, difference, product, and quotient of rational expressions with integral exponents of degree one and two (or higher).</p>	<p>2A.6K Determine the asymptotic restrictions on the domain of a rational function & represent domain and range using interval notation, inequalities, and set notation.</p>

NOTE: AP readiness may necessitate extensions from TEKS.

<p>Focus on Disciplinary Literacy</p> 	<p>Mathematical Process Standard (F) – Analyze mathematical relationships to connect and communicate mathematical ideas.</p>
	<p>Mathematical Process Standard (G) – Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.</p>

LEARNING SUPPORTS BY LESSON

There is a checkmark for the math support if the lesson	Lessons →	L1	L2	L3	L4	L5	L6	L7	L8
	Math Supports								
makes a connection to prior content or from a previous unit or academic year	Access Prior Knowledge	✓	✓	✓	✓	✓	✓	✓	✓
uses familiar contexts or experiences to make the learning relevant to students	Real-World Connections	✓	✓	✓	✓				
makes use of graphic organizers	Graphic Organizers	✓			✓		✓		✓
includes tools like rulers, protractors, patty paper, algebra tiles, etc.	Tools or Manipulatives								
incorporates tables, reference charts, displays, pictures, models, or color-coding	Visual Aids			✓		✓			
includes definitions, examples vs. nonexamples, cognates, etc.	Vocabulary Supports	✓	✓	✓	✓	✓		✓	
includes strategies that support language development									
asks students to discuss with their partner to prepare for whole class discussion	- Turn and Talk	✓	✓				✓	✓	
teacher facilitates a whole class discussion to debrief key learnings	- Guided Discussion	✓	✓	✓	✓	✓	✓	✓	✓
asks students to think independently, test their idea with a partner, and share whole group	- Think, Pair, Share			✓	✓	✓			
includes sentence stems to support students with explanations	- Sentence Stems								
provides opportunities for students to work with a partner or a group	Peer Collaboration	✓	✓	✓	✓	✓	✓	✓	✓
uses mnemonics such as SohCahToa	Mnemonics								
includes websites or equipment that enhances the lesson	Technological Support			✓	✓	✓			
content can be presented in different forms									
uses hands-on tools or manipulatives to represent the math	- Concrete								
uses drawings to represent the math	- Pictorial	✓	✓	✓	✓	✓	✓		
uses numbers and number sentences to represent the math	- Abstract	✓	✓	✓	✓	✓	✓	✓	✓

The EFFL Model

Before You EFFL!

Here are helpful resources that you guide you in the right direction before your EFFL lesson!

Why Should We EFFL?

The article advocates for the Experience First, Formalize Later (EFFL) teaching model, emphasizing its effectiveness in fostering deep understanding and flexible thinking in students. The author compares traditional teaching to a game of "Simon Says," where students merely mimic instructions without grasping underlying concepts. In contrast, EFFL encourages students to engage actively with problems, enhancing their ability to understand and apply math concepts creatively.

Tips for Lesson Planning

The article offers practical advice for effective lesson planning beyond the exhaustive and overly detailed approaches often emphasized during teacher training. It underscores the importance of thoughtful preparation but rejects the notion that teachers need to script every minute or detail of a class session.

Making the Most of Your EFFL Lesson Debrief

The article discusses the significance of the debriefing phase in the Experience First, Formalize Later (EFFL) lesson model, emphasizing its role in reinforcing learning and highlighting student contributions. The debrief session is seen as crucial for integrating academic vocabulary, emphasizing key lesson understandings, and valuing students' mathematical insights.

While You EFFL!

While each lesson may be unique in context and skills, all lessons benefit from the following practices:

Teacher Look Fors:

- Utilizing the Do Now to spark students' interest in the Activity.
- Use questioning to promote small group discussion and exploration, guided by monitoring questions.
- Connects Experience First to formal concepts using a **colored pencil/pen** to take notes along the margin during the Debrief.
- Facilitates whole-class discussions for students to reflect, share insights, and provides feedback that reinforces key concepts.
- Tracks time to adapt lesson pacing and support based on student response and engagement.

Students Look Fors:

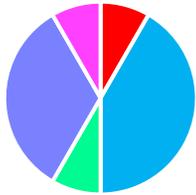
- In the Activity, students engage in group work and discourse.
- Exploring the activity, testing hypotheses and approaches (trial & error).
- Take notes on key ideas and concepts using different **colored pencil/pen** to take notes along the margin.
- Share thoughts and ideas that demonstrate their approach to their work.

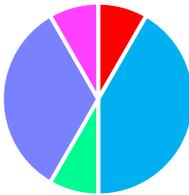
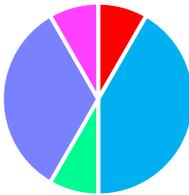
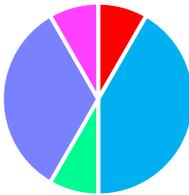
Other considerations

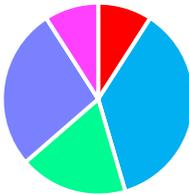
- During the **Experience First** phase, if most of your students seem stuck or disengaged, take a moment to pause, reset, and provide clear instructions. Some problems of the Activity are more suitable to do a whole-class discussion as a means to save some instructional time for Student Practice or the Exit Ticket. You are encouraged to adapt the EFFL (Experience First, Formalize Later) process to meet your students' needs while maintaining a focus on student-centered instruction.

ROADMAP

AT A GLANCE: Unit 9 – Rational Functions and Equations			
Day	Date	Lesson	Lesson Title
1		1	Inverse Variation Models
2		2	Combined Variation
3			<i>Unit 9 Success Day 1 – Review & Reteach topics from 9.1 – 9.2</i>
4		3	Intro to Rational Functions
5		4	Graphs of Rational Functions
6		5	Key Features of Graphs of Rational Functions
7			<i>Unit 9 Success Day 2 – Review & Reteach topics from 9.3 – 9.5</i>
8		6	Adding and Subtracting Rational Functions
9		7	Multiplying and Dividing Rational Functions
10		8	Solving Rational Functions
11			<i>Unit 9 Success Day 3 – Use as needed based on your data</i>
12			<i>Unit 9 Success Day 4 – Unit Assessment Review</i>
13			End of Unit 9 Assessment

Date: _____		
Lesson 1: Inverse Variation Models		
Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
<p>◆ 2A.6L Formulate and solve equations involving inverse variation (and combined variation).</p>	<p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> ▪ SE workbook ▪ graphing calculator 	<p><u>Look for teachers to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Connect students to their work with direct variation in Algebra 1 to inverse variation. <input type="checkbox"/> Focus students on the correct way to set up inverse variations with the known information to find the constant, k, first. <p><u>Look for students to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Find the constant of variation. <input type="checkbox"/> Use inverse variation to solve contextual problems.
	<div style="border: 1px solid #0070c0; padding: 5px;"> <p>Lesson Structure:</p> <ul style="list-style-type: none"> Do Now (5 min) INM (25 min) Debrief (5 min) Student Practice (20 min) Exit Ticket (5 min)  </div> <p>Mathematical Goal of this Lesson In this lesson, students will formulate, apply, and solve inverse variation models.</p> <p>Opportunities to CFU</p> <ul style="list-style-type: none"> ✓ INM #1-2 ✓ Debrief ✓ Student Practice #2-5 	
Important Vocabulary	<p>Other Notes to Inform Your Planning This lesson introduces students to inverse variation, but students may remember variation from Algebra 1 where we use direct variation when discussing linear functions.</p> <p>This is the first lesson in the Rational Functions and Equations unit. Students are exploring direct and inverse variation in order to understand that an inverse variation will always be written in the form $y = \frac{k}{x}$ where k is the constant of variation. Students will use this understanding moving forward to do combined variations, graph, and solve rational functions.</p>	
<ul style="list-style-type: none"> ▪ direct variation ▪ inverse variation ▪ constant of variation, k 		

Date: _____						
Lesson 2: Combined Variation						
Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors				
<p>◆ 2A.6L Formulate and solve equations involving inverse variation (and combined variation).</p>	<p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> ▪ SE workbook ▪ graphing calculator 	<p><u>Look for teachers to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Connect students to their work with inverse variation in the prior lesson. <input type="checkbox"/> Focus students on the correct way to set up combined variations with the proper direct and inverse variations in the scenario. Write the formula first. <p><u>Look for students to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Find the constant of variation in combined variation problems. <input type="checkbox"/> Use combined variation to solve contextual problems. 				
	<p>Lesson Structure:</p> <table border="1"> <tr> <td>■ Do Now (5 min)</td> <td rowspan="5">  </td> </tr> <tr> <td>■ INM (25 min)</td> </tr> <tr> <td>■ Debrief (5 min)</td> </tr> <tr> <td>■ Student Practice (20 min)</td> </tr> <tr> <td>■ Exit Ticket (5 min)</td> </tr> </table>		■ Do Now (5 min)		■ INM (25 min)	■ Debrief (5 min)
■ Do Now (5 min)						
■ INM (25 min)						
■ Debrief (5 min)						
■ Student Practice (20 min)						
■ Exit Ticket (5 min)						
<p>Mathematical Goal of this Lesson</p> <p>In this lesson, students will formulate, apply, and solve combined variation models.</p>	<p>Opportunities to CFU</p> <ul style="list-style-type: none"> ✓ After the INM ✓ Debrief <p style="text-align: right;">✓ Student Practice #1-4</p>					
<p>Important Vocabulary</p> <ul style="list-style-type: none"> ▪ direct variation ▪ inverse variation ▪ constant of variation, k ▪ joint variation ▪ combined variation 	<p>Other Notes to Inform Your Planning</p> <p>Combined variation is sometimes called joint variation, but it can use both direct and inverse variations as well as be raised to a power (students use combined variations in Pre-Calculus).</p> <p>Students use their understanding to determine solutions in real world problems and to write functions given various information. Students will use this understanding moving forward to graph and solve rational functions.</p>	<p>Student Know/Do Chart</p> <p>Students can</p> <p> Formulate, apply, and solve combined variation models.</p> <p> Direct and inverse variations are used in the same equation to create a combined variation. The direct variation creates the numerator with k and the inverse variation creates the denominator.</p>				

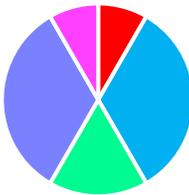
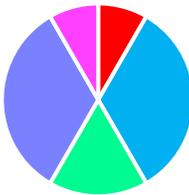
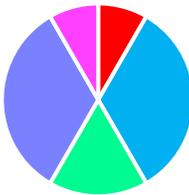
Date: _____		
Lesson 3: Intro to Rational Functions		
Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
<p>◆ 2A.6H Formulate rational equations that model real-world situations.</p>	<p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> ▪ SE workbook ▪ graphing calculator 	<p><u>Look for teachers to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Use real-world life experiences to connect students with rational functions. <input type="checkbox"/> Use guiding questions on the INM to keep the students centered in the lesson. <p><u>Look for students to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Explain in context how the function is affected by changing the number of people on the plan. <input type="checkbox"/> Interpret end behavior in context using the horizontal asymptote.
	<div style="border: 1px solid #0070c0; padding: 5px;"> <p>Lesson Structure:</p> <ul style="list-style-type: none"> Do Now (5 min) INM (20 min) Debrief (10 min) Student Practice (15 min) Exit Ticket (5 min)  </div> <p>Mathematical Goal of this Lesson In this lesson, students will write and evaluate rational functions. Students will also interpret horizontal asymptotes in a real-world context.</p> <p>Opportunities to CFU</p> <ul style="list-style-type: none"> ✓ INM problem #3 & 6 ✓ Student Practice #4-6 ✓ Debrief 	
Important Vocabulary		Student Know/Do Chart
<ul style="list-style-type: none"> ▪ rational expression ▪ rational parent function, $f(x) = \frac{1}{x}$ ▪ horizontal asymptote 	<p>Other Notes to Inform Your Planning We want students to begin thinking about what a rational function is and why we might need them. To do this, we are going to focus on a rational function in context. To start the lesson, ask if anyone can explain what a cell phone "Family" or group plan is. This is our context for the lesson today. You are interested in starting a group plan so that you can save money each month. As you add more people to the plan, what happens to the price per person. Have the groups work together to complete the entire INM.</p>	<p style="background-color: #ffff00; padding: 2px;">Focus on Disciplinary Literacy</p> <div style="border: 1px solid black; padding: 5px; display: flex; align-items: center;">  <div style="margin-left: 10px;">INM #4-7</div> </div> <p>Students can</p> <p> Write and evaluate rational functions and interpret horizontal asymptotes in context.</p> <p> A rational function is the ratio of two polynomial functions, $p(x)$ and $q(x)$ in the form $f(x) = \frac{p(x)}{q(x)}$.</p> <p> The horizontal asymptote of a rational function determines the end behavior.</p>

Date: _____		
Lesson 4: Graphs of Rational Functions		
<p>Standard(s)</p> <p>◆ 2A.2A Graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^2$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = x$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.</p> <p>◆ 2A.6G Analyze the effect on the graph of $f(x) = 1/x$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d.</p>	<p>Notes for Intellectual Preparation & Lesson Planning</p> <p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> SE workbook graphing calculator or Desmos <div data-bbox="499 342 1251 618" style="border: 1px solid black; padding: 5px;"> <p>Lesson Structure:</p> <ul style="list-style-type: none"> ■ Do Now (5 min) ■ INM (20 min) ■ Debrief (10 min) ■ Student Practice (15 min) ■ Exit Ticket (5 min)  </div> <p>Mathematical Goal of this Lesson</p> <p>In this lesson, students will describe how a function in the form of $y = \frac{a}{x-h} + k$ has been transformed from the parent function $y = \frac{1}{x}$. Students will also identify key attributes of the graphs of rational functions in the form of $y = \frac{a}{x-h} + k$.</p> <p>Opportunities to CFU</p> <ul style="list-style-type: none"> ✓ INM #4 ✓ Debrief ✓ Student Practice #1-4 	<p>Lesson Look Fors</p> <p><u>Look for teachers to...</u></p> <ul style="list-style-type: none"> ☐ Challenge students to graph the families of graphs in the INM without technology. ☐ Remind students the previous transformations they have performed to help them explore rational functions. <p><u>Look for students to...</u></p> <ul style="list-style-type: none"> ☐ Perform a variety of transformations of rational functions and discuss with their table mates the attributes of the graph and how the transformation affected them.
<p>Important Vocabulary</p> <ul style="list-style-type: none"> rational parent function, $f(x) = \frac{1}{x}$ horizontal asymptote vertical asymptote hyperbola 	<p>Other Notes to Inform Your Planning</p> <p>Today we will be looking at the most basic rational function, $y = \frac{1}{x}$. We want to focus on the graph of this function and identify key features of it, including the asymptotes, domain, and range. We also will use our understanding of how we can transform functions using $y = af(x - h) + k$ as we learned way back in Unit 1. Our activity today will start with a reminder of how we can transform parent functions. Students should work in groups to complete the entire activity. Each person will need two different colors to graph the functions. It's recommended that that complete the INM without using a graphing calculator or Desmos but they could use it to check their work once they have made a prediction about what the graphs will look like. The intention is that students will remember how to transform a function as they work with function families that they are already familiar with and then they can apply this to the function $y = \frac{1}{x}$.</p> <div data-bbox="1062 984 1444 1122" style="border: 1px solid black; padding: 5px;"> <p style="background-color: yellow; text-align: center;">Focus on Disciplinary Literacy</p> <div style="display: flex; align-items: center; justify-content: center;">  <div> <p style="text-align: center;">Student Practice #4</p> </div> </div> </div>	<p>Student Know/Do Chart</p> <p>Students can</p> <ul style="list-style-type: none"> 🎯 Describe how the rational parent function has been transformed when a rational function is in the form $y = \frac{a}{x-h} + k$. 🎯 Identify key attributes of a given rational function's graph. 📖 When a rational function is in the form $y = \frac{a}{x-h} + k$, a controls vertical stretch/shrink, h controls horizontal shift, and k controls the vertical shift.

Date: _____		
Lesson 5: Key Features of Graphs of Rational Functions		
Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
<p>◆ 2A.6K Determine the asymptotic restrictions on the domain of a rational function & represent domain and range using interval notation, inequalities, and set notation.</p> <p>◆ 2A.6G Analyze the effect on the graph of $f(x) = 1/x$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d.</p>	<p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> SE workbook graphing calculator or Desmos <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Lesson Structure:</p> <ul style="list-style-type: none"> Do Now (5 min) INM (20 min) Debrief (10 min) Student Practice (15 min) Exit Ticket (5 min)  </div> <p>Mathematical Goal of this Lesson</p> <p>In this lesson, students will find the zeros of the numerator of a rational function to find the x-intercepts and the zeros of the denominator to determine limits on the domain of the function. Students will be able to identify the asymptotes and/or holes for the graphs of rational functions by inspecting the limits on the domain.</p>	<p>Look for teachers to...</p> <ul style="list-style-type: none"> Support students in graphing rational functions using Desmos to explore discontinuities and how the zeros of the numerator behave in the graph. <p>Look for students to...</p> <ul style="list-style-type: none"> Graph multiple rational functions in factored form. Identify the point discontinuities and asymptotes and any x-intercepts of the curves by exploring the zeros of the numerator and denominator.
	<p>Important Vocabulary</p> <ul style="list-style-type: none"> rational function point of discontinuity continuous graph removeable discontinuity non-removeable discontinuity discontinuous graph horizontal asymptote vertical asymptote 	<p>Opportunities to CFU</p> <ul style="list-style-type: none"> ✓ INM problems #1-5 ✓ Debrief ✓ Student Practice #1-5 <p>Other Notes to Inform Your Planning</p> <p>We will be looking at rational functions that have x's in both the numerator and denominator. Students will investigate how the zeros of the numerator and denominator show up in the graph of the function. Have students work together in groups to complete the entire activity. We'd recommend that they use Desmos to graph the functions. The asymptotes can show up weirdly on graphing calculators.</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px; text-align: center;"> <p>Focus on Disciplinary Literacy</p>  <p>INM & Debrief</p> </div>

Date: _____		
Lesson 6: Adding and Subtracting Rational Functions		
Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
<p>◆ 2A.7F Determine the sum, difference, product, and quotient of rational expressions with integral exponents of degree one and two (or higher).</p>	<p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> SE workbook graphing calculator <div data-bbox="499 375 1251 651" style="border: 1px solid black; padding: 5px;"> <p>Lesson Structure:</p> <ul style="list-style-type: none"> ■ Do Now (5 min) ■ INM (15 min) ■ Debrief (10 min) ■ Student Practice (20 min) ■ Exit Ticket (5 min)  </div> <p>Mathematical Goal of this Lesson In this lesson, students will simplify rational functions using addition and subtraction.</p> <p>Opportunities to CFU</p> <ul style="list-style-type: none"> ✓ INM #8-20 ✓ Debrief ✓ Student Practice #1-4 	<p>Look for teachers to...</p> <ul style="list-style-type: none"> <input type="checkbox"/> Remind students to factor all terms first and have them work in factored form as long as possible. <input type="checkbox"/> Focus students on the denominators and how to get common denominators by focusing on the concept not just the procedure. <p>Look for students to...</p> <ul style="list-style-type: none"> <input type="checkbox"/> Find the least common denominator so they can add or subtract rational expressions. <input type="checkbox"/> Simplify their answers.
Important Vocabulary	<p>Other Notes to Inform Your Planning Today we are learning about simplifying, adding and subtracting rational expressions. We started in the Do Now. Riley is in 5th grade and is learning about fractions. Students began in the Do Now by trying Riley's homework, reducing, adding, and subtracting fractions. Each problem showcases an important idea about the operations with fractions. Start the INM by having students work with their table partners to answer INM #1 based on their Do Now. You may want to pause first and debrief the Do Now problems with the margin notes provided below and discuss INM #1 before moving on.</p> <div data-bbox="1167 919 1444 1084" style="border: 1px solid black; padding: 5px; text-align: center;"> <p>Focus on Disciplinary Literacy</p>  <p>INM</p> </div>	<p>Student Know/Do Chart</p> <p>Students can</p> <ul style="list-style-type: none">  Simplify rational functions using addition and subtraction  Adding and subtracting rational expressions requires getting the terms to have a common denominator first by multiplying the numerator and denominator by missing terms to get the least common denominator.
<ul style="list-style-type: none"> rational expression factor (verb, noun) greatest common factor undefined reciprocal divisor complex fraction least common multiple (LCM) least common denominator (LCD) 		

Date: _____												
Lesson 7: Multiplying and Dividing Rational Functions												
Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors										
<p>◆ 2A.7F Determine the sum, difference, product, and quotient of rational expressions with integral exponents of degree one and two (or higher).</p>	<p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> ▪ SE workbook ▪ graphing calculator <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Lesson Structure:</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 20px; text-align: center;">■</td> <td>Do Now (5 min)</td> </tr> <tr> <td style="text-align: center;">■</td> <td>INM (15 min)</td> </tr> <tr> <td style="text-align: center;">■</td> <td>Debrief (10 min)</td> </tr> <tr> <td style="text-align: center;">■</td> <td>Student Practice (20 min)</td> </tr> <tr> <td style="text-align: center;">■</td> <td>Exit Ticket (5 min)</td> </tr> </table>  </div> <p>Mathematical Goal of this Lesson In this lesson, students will simplify rational functions using multiplication and division.</p> <p>Opportunities to CFU</p> <ul style="list-style-type: none"> <li style="width: 50%;">✓ INM <li style="width: 50%;">✓ Student Practice #1-4 <li style="width: 50%;">✓ Debrief <p>Other Notes to Inform Your Planning</p> <p>In the last lesson, students used their knowledge about reducing, adding and subtracting fractions to reduce, add, and subtract rational functions. Today they will be taking their knowledge about multiplying and dividing fractions and applying it to rational functions. To begin, students can work on #1 in their groups to summarize what they did in the Do Now. Like yesterday, you could pause after the Do Now and #1 to debrief and review before moving on to the rational functions but it isn't a must. Students should be able to complete #2 without it. As students are working on #2, direct them back to their work in the Do Now as much as possible when they get stuck. Ask them to explain how they solved the problems when they were working with fractions. This activity goes quickly.</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px; text-align: center;"> <p>Focus on Disciplinary Literacy</p>  <p>INM & Debrief</p> </div>	■	Do Now (5 min)	■	INM (15 min)	■	Debrief (10 min)	■	Student Practice (20 min)	■	Exit Ticket (5 min)	<p>Lesson Look Fors</p> <p><u>Look for teachers to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Encourage students to connect what they know from fraction multiplication and division to rational expressions. <input type="checkbox"/> Discuss simplification of final products and how the class should leave answers (factored or distributed). <p><u>Look for students to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Factor the numerator and denominator to determine if there are common factors. <input type="checkbox"/> Convert division into multiplication and simplify out common factors in their final answer.
	■	Do Now (5 min)										
■	INM (15 min)											
■	Debrief (10 min)											
■	Student Practice (20 min)											
■	Exit Ticket (5 min)											
Important Vocabulary		Student Know/Do Chart										
<ul style="list-style-type: none"> ▪ rational expression ▪ factor (verb, noun) ▪ greatest common factor ▪ undefined ▪ reciprocal ▪ divisor 		<p>Students can</p> <p> Simplify rational functions using multiplication and division.</p> <p> Division of rational expressions is equivalent to multiplication by the reciprocal $\left(\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}\right)$.</p> <p> To be simplified, a product should have the numerator and denominator factored and then any common factors simplified using the multiplicative identity $\left(\frac{a}{a} = 1\right)$.</p>										

Date: _____													
Lesson 8: Solving Rational Functions													
Standard(s) ◆ 2A.6I Solve rational equations that have real solutions. ◆ 2A.6J Determine the reasonableness of a solution to a rational equation.	Notes for Intellectual Preparation & Lesson Planning Necessary Materials and Pre-Lesson Prep <ul style="list-style-type: none"> SE workbook graphing calculator <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> Lesson Structure: <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 20px; text-align: center;">■</td> <td>Do Now (5 min)</td> <td rowspan="5" style="text-align: center; vertical-align: middle;"></td> </tr> <tr> <td style="text-align: center;">■</td> <td>INM (20 min)</td> </tr> <tr> <td style="text-align: center;">■</td> <td>Debrief (10 min)</td> </tr> <tr> <td style="text-align: center;">■</td> <td>Student Practice (20 min)</td> </tr> <tr> <td style="text-align: center;">■</td> <td>Exit Ticket (5 min)</td> </tr> </table> </div> Mathematical Goal of this Lesson In this lesson, students will solve equations with rational functions using a variety of methods and identify any extraneous solutions.	■	Do Now (5 min)		■	INM (20 min)	■	Debrief (10 min)	■	Student Practice (20 min)	■	Exit Ticket (5 min)	Lesson Look Fors Look for teachers to... <ul style="list-style-type: none"> <input type="checkbox"/> Support students as they solve a variety of rational equations. <input type="checkbox"/> Make sure students are checking their solutions Look for students to... <ul style="list-style-type: none"> <input type="checkbox"/> Solve a variety of problems using simple and more complex rational expressions. Students need to be able to explain their methods. <input type="checkbox"/> Check their solutions and determine if any are extraneous.
■	Do Now (5 min)												
■	INM (20 min)												
■	Debrief (10 min)												
■	Student Practice (20 min)												
■	Exit Ticket (5 min)												
Important Vocabulary <ul style="list-style-type: none"> rational equations extraneous solutions 	Opportunities to CFU <ul style="list-style-type: none"> ✓ INM ✓ Debrief <p style="text-align: right;">✓ Student Practice #1-3</p> Other Notes to Inform Your Planning Once again, we're helping Riley and Xavier with their homework. The goal is that as students solve the problems in Riley's homework, they will note the process they're using and will apply it to Xavier's homework. The problems were paired according to the strategy used to solve the question. For example, in the first question of Riley's homework, the denominators are the same, so we just need to solve the numerator. This is the same in Xavier's homework. Give groups plenty of time to just work through the problems. There isn't one set way to solve each problem so let them try out different methods. The intention of today's lesson is that students use a variety of methods to solve depending on what they notice in the structure of the equations.	Student Know/Do Chart Students can <ul style="list-style-type: none">  Solve equations with rational functions using a variety of methods.  Determine if any solutions for a rational equation are extraneous.  There can be different methods to solve a rational equation, but any of them could cause extraneous solutions so we must check our answers in the original equation. 											
<div style="border: 1px solid black; padding: 5px; display: inline-block;"> Focus on Disciplinary Literacy  INM & Debrief </div>													

Recommended Unit 9 Success Days Material and Resources

Date: _____

To review variation and transformations of rational functions:

GeoGebra Interactive Applets:

- Exploring Inverse Variation
- Direct and Inverse Variation
- Transforming Rational Functions

Sample Activities & Tasks:

- TI-Calculator Activity: Inverse Variation

Content Video Lessons:

- Introduction to Direct & Inverse Variation
- Recognizing Direct & Inverse Variation
- Recognizing Direct & Inverse Variation: Table
- Inverse Variation Word Problem
- Joint and Combined Variation
- Transforming Rational Functions
- Attributes of Rational Functions

To review operations of rational expressions:

GeoGebra Interactive Applets:

- Rational Expressions
- Simplifying Rational Expressions

Desmos Interactive Applets:

- Multiplying & Simplifying Rational Expressions
- Adding and Subtracting Rational Expressions

Content Video Lessons:

- Simplifying Rational Expressions
- Multiplying & Dividing Rational Expressions, Part 1
- Multiplying & Dividing Rational Expressions, Part 2
- Multiplying & Dividing Rational Expressions, Part 3
- Adding and Subtracting Rational Expressions with Like Denominators
- Finding the Least Common Multiple of Polynomials
- Simplifying Complex Fractions

To review graphs of rational functions and their discontinuities:

GeoGebra Interactive Applets:

- Holes and Asymptotes
- Exploring Vertical Asymptotes
- Graphing Rational Functions

Sample Activities & Tasks:

- TI-Calculator Activity: Graphs of Rational Functions

Content Video Lessons:

- Finding Horizontal and Vertical Asymptotes
- Finding Removeable Discontinuities
- Graphing Rational Functions

Date: _____

Unit 9 Exam

Standard(s)	Notes for Intellectual Preparation & Lesson Planning
<p>◆ 2A.2A Graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^2$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = x$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval. Problems #6, 7, 10abd</p> <p>◆ 2A.6I Solve rational equations that have real solutions. Problem #3</p> <p>◆ 2A.6L Formulate and solve equations involving inverse variation (and combined variation). Problems #9</p> <p>◆ 2A.7F Determine the sum, difference, product, and quotient of rational expressions with integral exponents of degree one and two (or higher). Problems #1, 5</p> <p>◆ 2A.6G Analyze the effect on the graph of $f(x) = 1/x$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d. Problem #4</p> <p>◆ 2A.6H Formulate rational equations that model real-world situations. Problem #8</p> <p>◆ 2A.6J Determine the reasonableness of a solution to a rational equation. Problem #3</p> <p>◆ 2A.6K Determine the asymptotic restrictions on the domain of a rational function & represent domain and range using interval notation, inequalities, and set notation. Problems #2, 10c</p>	<p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none">▪ Algebra 2 Unit 9 Exam▪ Assessment Companion for Algebra 2 Unit 9 Exam found on Curriculum Corner <p>Notes to Inform Your Planning</p> <p>Review the Unit 9 Exam on Curriculum Corner. Internalize & create an exemplar for the assessment prior to teaching the unit as part of unpacking the unit. Use the exemplar to spar with the answer key provided on Curriculum Corner.</p> <p>Administer the Unit 9 Exam following guidance on the Scope & Sequence to ensure the scoring deadline is met.</p>

UNPACKED STANDARDS

Focus standards for this unit.

Standards Clarification		
Standards	Specificity	Notes/Explanations/Examples
<p>2A.2A Graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^2$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = x$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.</p>	<p>Graph THE FUNCTIONS $f(x) = \frac{1}{x}$</p> <p>Including, but not limited to:</p> <ul style="list-style-type: none"> • Representations of functions, including graphs, tables, and algebraic generalizations <ul style="list-style-type: none"> ○ Rational (reciprocal of x), $f(x) = \frac{1}{x}$ • Connections between representations of families of functions • Comparison of similarities and differences of families of functions <p>Analyze Including, but not limited to:</p> <ul style="list-style-type: none"> • Domain and range of the function <ul style="list-style-type: none"> ○ Domain – set of input values for the independent variable over which the function is defined ○ Range – set of output values for the dependent variable over which the function is defined ○ Representation for domain and range ○ Domain and range of the function versus domain and range of the contextual situation • Key attributes of functions <ul style="list-style-type: none"> ○ Intercepts/Zeros ○ Symmetries ○ Asymptotic behavior – behavior such that as x approaches infinity, $f(x)$ approaches a given value • Use key attributes to recognize and sketch graphs • Application of key attributes to real-world problem situations 	<ul style="list-style-type: none"> • Grade Level(s): <ul style="list-style-type: none"> ○ The notation \mathbb{R} represents the set of real numbers, and the notation \mathbb{Z} represents the set of integers. ○ Algebra I studied parent functions $f(x) = x$, $f(x) = x^2$, and $f(x) = b^x$ and their key attributes. ○ Precalculus will study polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions. ○ Various mathematical process standards will be applied to this student expectation as appropriate.
<p>2A.6I Solve rational equations that have real solutions.</p>	<p>Including, but not limited to:</p> <ul style="list-style-type: none"> • Rational equations composed of linear or quadratic functions • Limited to real solutions • Methods for solving rational equations with and without technology <ul style="list-style-type: none"> ○ Graphs ○ Algebraic methods ○ Justifications of solutions with and without technology <p>Real-world problem situations modeled by rational functions</p> <ul style="list-style-type: none"> ○ Justification of reasonableness of solutions in terms of real-world problem situations or data collections 	<ul style="list-style-type: none"> • Grade Level(s): <ul style="list-style-type: none"> ○ Algebra II introduces the rational equation and its applications. ○ Various mathematical process standards will be applied to this student expectation as appropriate.

Standards Clarification

Standards	Specificity	Notes/Explanations/Examples
2A.6L Formulate and solve equations involving inverse variation (and combined variation).	Formulate Including, but not limited to: <ul style="list-style-type: none"> • Characteristics of variation <ul style="list-style-type: none"> ○ Constant of variation ○ Particular equation to represent variation ○ Types of variation • Real-world problem situations involving variation • Reasonableness of solutions mathematically and in context of real-world problem situations Solve Including, but not limited to: <ul style="list-style-type: none"> • Methods for solving variation equations with and without technology <ul style="list-style-type: none"> ○ Graphs ○ Algebraic methods ○ Justification of solutions with and without technology Real-world problem situations modeled by rational functions <ul style="list-style-type: none"> ○ Justification of reasonableness of solutions in terms of real-world problem situations or data collections 	<ul style="list-style-type: none"> • Grade Level(s): <ul style="list-style-type: none"> ○ Prior grade levels studied direct variation and proportionality. ○ Algebra II introduces inverse variation and its applications in problem situations. ○ Precalculus will continue to investigate rational functions. ○ Various mathematical process standards will be applied to this student expectation as appropriate.
2A.7F Determine the sum, difference, product, and quotient of rational expressions with integral exponents of degree one and two (or higher).	Including, but not limited to: <ul style="list-style-type: none"> • Equivalent rational expressions <ul style="list-style-type: none"> ○ Multiplication by a fractional form of 1 • Simplification of rational expressions <ul style="list-style-type: none"> ○ Factorization of numerator and denominator ○ Cancellation or dividing out of common factors • Operations with rational expressions <ul style="list-style-type: none"> ○ Sum and difference ○ Product and quotient 	<ul style="list-style-type: none"> • Grade Level(s): <ul style="list-style-type: none"> ○ Previous grade levels simplified and performed operations on fractions. ○ Algebra II simplifies and performs operations on rational expressions involving variables. ○ Various mathematical process standards will be applied to this student expectation as appropriate.
2A.6G Analyze the effect on the graph of $f(x) = 1/x$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a , b , c , and d .	Including, but not limited to: <ul style="list-style-type: none"> • General form of the rational function <ul style="list-style-type: none"> ○ Rational function • Representations with and without technology <ul style="list-style-type: none"> ○ Graphs ○ Tables ○ Verbal descriptions ○ Algebraic generalizations • Effects on the graph of $f(x) = \frac{1}{x}$, when parameters a, b, c, and d are changed in $f(x) = a\left(\frac{1}{b(x-c)}\right) + d$ or $f(x) = \frac{a}{b(x-c)} + d$ <ul style="list-style-type: none"> ○ Effects on the graph of $f(x) = \frac{1}{x}$, when $f(x)$ is replaced by $af(x)$ with and without technology ○ Effects on the graph of $f(x) = \frac{1}{x}$, when $f(x)$ is replaced by $f(bx)$ with and without technology 	<ul style="list-style-type: none"> • Grade Level(s): <ul style="list-style-type: none"> ○ Algebra I determined effects on the graphs of the parent functions, $f(x) = x$ and $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d. ○ Algebra II introduces the rational function and its transformations. ○ Various mathematical process standards will be applied to this student expectation as appropriate.

Standards Clarification

Standards	Specificity	Notes/Explanations/Examples
	<ul style="list-style-type: none"> ○ Effects on the graph of $f(x) = \frac{1}{x}$, when $f(x)$ is replaced by $f(x - c)$ with and without technology ○ Effects on the graph of $f(x) = \frac{1}{x}$, when $f(x)$ is replaced by $f(x) + d$ with and without technology ● Connections between the critical attributes of transformed function and $f(x) = \frac{1}{x}$ <ul style="list-style-type: none"> ○ Determination of parameter changes given a graphical or algebraic representation ○ Determination of a graphical representation given the algebraic representation or parameter changes ○ Determination of an algebraic representation given the graphical representation or parameter changes ● Descriptions of the effects on the domain and range by the parameter changes ● Descriptions of the effects on the asymptotes by the parameter changes ● Effects of multiple parameter changes <ul style="list-style-type: none"> ○ Mathematical problem situations ○ Real-world problem situation 	
2A.6H Formulate rational equations that model real-world situations.	Including, but not limited to: <ul style="list-style-type: none"> ● Rational equations composed of linear or quadratic functions ● Data collection activities with and without technology <ul style="list-style-type: none"> ○ Data modeled by rational functions ● Real-world problem situations <ul style="list-style-type: none"> ○ Real-world problem situations modeled by rational functions ● Data tables ● Technology methods <ul style="list-style-type: none"> ○ Transformations of $f(x) = \frac{1}{x}$ 	<ul style="list-style-type: none"> ● Grade Level(s): <ul style="list-style-type: none"> ○ Algebra II introduces the rational equation and its applications. ○ Various mathematical process standards will be applied to this student expectation as appropriate.
2A.6J Determine the reasonableness of a solution to a rational equation.	Including, but not limited to: <ul style="list-style-type: none"> ● Rational equations composed of linear or quadratic functions ● Justification of solutions to rational equations with and without technology <ul style="list-style-type: none"> ○ Verbal description ○ Tables ○ Graphs ○ Substitution of solutions into original functions ● Justification of reasonableness of solutions in terms of real-world problem situations or data collections 	<ul style="list-style-type: none"> ● Grade Level(s): <ul style="list-style-type: none"> ○ Algebra II introduces rational equations and solving rational equations. ○ Various mathematical process standards will be applied to this student expectation as appropriate.

Standards Clarification

Standards	Specificity	Notes/Explanations/Examples
<p>2A.6K Determine the asymptotic restrictions on the domain of a rational function & represent domain and range using interval notation, inequalities, and set notation.</p>	<p>Including, but not limited to:</p> <ul style="list-style-type: none"> • Discontinuity in rational functions <ul style="list-style-type: none"> ○ Discontinuity – characteristic of a function where it is not continuous at some point along its graph; place where there is a break in the continuous curve of the function ○ Asymptote – a line that is approached and may or may not be crossed ○ Point (removable) discontinuity – hole or discontinuity in the graph of a rational function generated when a factor in the denominator, that should create a vertical asymptote, reduces out with an equivalent factor in the numerator • Graphical analysis using discontinuity <ul style="list-style-type: none"> ○ Domain and range ○ Analyzing graph of function in regions formed on graph <p>Including, but not limited to:</p> <ul style="list-style-type: none"> • Inequality notation <ul style="list-style-type: none"> ○ Ex: $x < 5$ or $x > 8$? ○ Ex: $-3 < y < 6$ ○ Ex: $x < -3$ or $0 < x < 2$ or $x > 4$ • Set notation <ul style="list-style-type: none"> ○ Ex: $\{x x \in, \mathbb{R}, x < 5 \text{ or } x > 8\}$ ○ Ex: $\{y y \in, \mathbb{R}, -3 < y < 6\}$ ○ Ex: $\{x x \in, \mathbb{R}, x < -3 \text{ or } 0 < x < 2 \text{ or } x > 4\}$ • Interval notation <ul style="list-style-type: none"> ○ Ex: $(-\infty, 5) \cup (8, \infty)$ ○ Ex: $(-3, 6)$ ○ Ex: $(-\infty, -3) \cup (0, 2) \cup (4, \infty)$ 	<ul style="list-style-type: none"> • Grade Level(s): <ul style="list-style-type: none"> ○ Algebra II introduces the rational function and its attributes. ○ Precalculus will continue to investigate rational functions. ○ Various mathematical process standards will be applied to this student expectation as appropriate.

VERTICAL STANDARDS

This section details the **progression** of key student expectations (SEs) in the courses **before** and **after** this course. This will help you understand what **prior knowledge skills to build upon** and guide you in knowing what **skills you are preparing your students** for in the subsequent course.

Algebra 1	Algebra 2	Pre-Calculus
<ul style="list-style-type: none"> • A.2D Write and solve equations involving direct variation. • A.7C Determine the effects on the graph of $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, and $f(bx)$ for specific values of a, b, c, and d. • A.10A Add and subtract polynomials of degree one and degree two. • A.10B Multiply polynomials of degree one and degree two. • A.10C Determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend. • A.10D Rewrite polynomial expressions of degree one and degree two in equivalent forms using the distributive property. 	<ul style="list-style-type: none"> • 2A.2A Graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = x$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval. • 2A.6G Analyze the effect on the graph of $f(x) = 1/x$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d. • 2A.6H Formulate rational equations that model real-world situations. • 2A.6I Solve rational equations that have real solutions. • 2A.6J Determine the reasonableness of a solution to a rational equation. • 2A.6K Determine the asymptotic restrictions on the domain of a rational function & represent domain and range using interval notation, inequalities, and set notation. • 2A.6L Formulate and solve equations involving inverse variation. • 2A.7F Determine the sum, difference, product, and quotient of rational expressions with integral exponents of degree one and two (or higher). 	<ul style="list-style-type: none"> • (2.G) Graph functions, including exponential, logarithmic, sine, cosine, rational, polynomial, and power functions and their transformations, including $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems. • (2.I) Determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum/ minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing. • (2.J) Analyze and describe end behavior of functions, including exponential, logarithmic, rational, polynomial, and power functions, using infinity notation to communicate this characteristic in mathematical and real-world problems. • (2.K) Analyze characteristics of rational functions and the behavior of the function around the asymptotes, including horizontal, vertical, and oblique asymptotes. • (2.L) Determine various types of discontinuities in the interval $(-\infty, \infty)$ as they relate to functions and explore the limitations of the graphing calculator as it relates to the behavior of the function around discontinuities. • (2.N) Analyze situations modeled by functions, including exponential, logarithmic, rational, polynomial, and power functions, to solve real-world problems. • (5.L) Solve rational inequalities with real coefficients by applying a variety of techniques and write the solution set of the rational inequality in interval notation in mathematical and real-world problems.